VOLUME 76 SEPARATE No. 20

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

MAY, 1950

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

THREE-DIMENSIONAL DISPLACEMENT DIAGRAMS FOR SPACE FRAME STRUCTURES

By Walter Worthington Ewell, Jun. ASCE

STRUCTURAL DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THREE-DIMENSIONAL DISPLACEMENT DIAGRAMS FOR SPACE FRAME STRUCTURES

By Walter Worthington Ewell,[1] Jun. ASCE

Synopsis

Because the Williot-Mohr displacement diagram has proved to be an inestimable aid in planar structure analysis, it is conceivable that a similar diagram applicable to space structures might also have some merit. The development of such a diagram, suitable for determining the displacement of any space frame joint along the three coordinate axes of space, has been the purpose of this paper.

Three-dimensional displacement diagrams have been presented for a simple space pedestal and a second-degree indeterminate space truss. The space pedestal displacement diagram illustrates the graphical method in its simpler aspects. The displacement diagram for the space truss circumvents none of the problems inherent in the method and, furthermore, demonstrates the use of a rotational correction diagram. Both graphical examples presented in this paper have been developed progressively in three separate phases to meliorate any fears of an overly complex problem.

Elementary principles of descriptive geometry are involved in the orthographic representation of the displacement diagram. These geometric principles should be thoroughly understood, however, before reading this paper.

All displacement values obtained graphically have been confirmed by least work analyses of both structures. The complete research, including analytical verifications, has been filed for reference in the Engineering Societies Library.[1a]

Introduction

Since its inception, the Williot-Mohr displacement diagram has gradually been adapted to the solution of many planar structural problems previously

Note.—Written comments are invited for publication; the last discussion should be submitted by October 1, 1950.

[1] Asst. Prof., The Johns Hopkins Univ., Baltimore, Md.

[1a] 29 W. 39th St., New York 18, N. Y.

managed with the tedious, unwieldy, and sometimes abstruse classical methods. The most important use of the displacement diagram, aside from the determination of the intrinsically important truss deflection, has been in the analysis of statically indeterminate structures. With this significant graphical method, the calculation of end moments at truss panel points has been simplified and the construction of influence lines for redundant reactions and redundant internal members has been facilitated. Other important applications of this displacement diagram have been suggested.[2] Today the Williot-Mohr diagram exists as a vital part of the information required by the structural engineer working with planar structures of all types. The applications of the diagram, although manifold, probably have not been exhausted.

Multitudes of space structures are in existence in the United States today. Framed pedestals, water tank towers, pipe line bridges, electric power transmission towers, and domes are only a few examples of the space frames that abound. Many more structures—undeniably space frames—have been designed as planar structures to avoid the tedium of a three-dimensional analysis. With the ever increasing use of welded joints manifestly facilitating the connections of members in space, and with the growing knowledge that many constructions can be more economically designed as space structures, the erection of true space frames should be even more prevalent in the future. As the demand for space structures increases, the desire for more rapid methods of analysis will also increase. Therefore, it is not presumptuous to assume that any additional information pertinent to the analysis of determinate and indeterminate space structures may some day be of benefit. This assumption motivated the development of the three-dimensional displacement diagram.

As is true in planar structural analysis, determinate and indeterminate space frames can be analyzed, and deflections can be computed by many of the classical methods involving work and strain energy principles.[3] One of the better systems for computing joint deflection is the virtual work method wherein a virtual or imaginary unit load is placed upon the structure in the direction of the desired deflection. The ensuing calculations for stress and deformation result in the desired deflection.[4] These calculations, particularly in space structure analysis, are inordinately long, even if only one deflection is desired. The computations, furthermore, increase in complexity if the space structure is indeterminate to any degree. If all components of displacement are required, the individual calculations must be performed three times at each joint.

Another method of analyzing the panel-point displacements within a space structure was presented in 1946.[5] The calculations for deflection involve the solution of as many simultaneous equations as there are unknown displacements within the structure. Actually, the solution for deflection is but one of the necessary steps in the solution for member stresses within the frame

[2] "Williot Equations for Statically Indeterminate Structures," by Charles A. Ellis, *Transactions*, ASCE, Vol. 100, 1935, p. 580.

[3] "Theory of Structures," by S. Timoshenko and D. H. Young, McGraw-Hill Book Co., Inc., New York, N. Y., 1945, p. 326.

[4] "Theory of Modern Steel Structures," by L. E. Grinter, Macmillan Co., New York, N. Y., 1937, Vol. II, p. 70.

[5] "Theory of Indeterminate Structures," by L. C. Maugh, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 300.

and is not meant to be a solution for deflection alone. This system can be used advantageously in the simpler space frames where few joints give rise to but few unknown displacements. The method becomes laborious as the required equations (to be solved simultaneously) increase in number. Consequently, although many decry the use of graphical methods because of the inaccuracies involved, it is evident that a workable and comparatively rapid method of finding space structure deflections in one graphical construction can lessen the lengthy computations encountered in space frame analysis.

The Space Displacment Diagram

Fundamentally the construction of the space displacement diagram involves the following steps:

1. The solution of stress or axial force in all members of the space frame, whether determinate or indeterminate in nature—axial forces are found by one of the analytical methods of analysis.
2. The computation of strain within each member of the frame—values of $\frac{PL}{AE}$ are computed and considered either elongations or contractions, dependent upon whether tension or compression exists in the member.
3. The construction of a plan and elevation view of the space frame concerned—these diagrams serve to ascertain slope of individual members.
4. The selection of a fixed point and fixed member with which to start the space displacement diagram—the fixed point and line are represented orthographically in plan and elevation views.
5. The construction of strain lines to some scale, of lengths, equal to the strains of those members emanating from the fixed point and fixed member—
   (*a*) These lines are represented orthographically in the plan and elevation views and their slopes are determined from the orthographic representation of the actual space frame.
   (*b*) Foreshortened strains are plotted where necessary, if the member itself is foreshortened in its orthographic representations.
6. The erection of planes perpendicular to the ends of the strain lines.
7. The intersection of the planes perpendicular to the ends of ths strain lines—
   (*a*) The simultaneous intersection of at least three planes is necessary for the determination of a point of intersection.
   (*b*) Where the three planes are perpendicular to a common plane, their intersection with a fourth plane may be required to afford a point of intersection.
8. The location of the displaced joint—the intersection of the three planes, represented orthographically in all views, will be the location of the displaced joint. (In a planar Williot diagram the directions of only two strains are required to determine the new location of a displaced joint. Perpendiculars at the ends of these strain lines representing arc lengths are intersected to locate the adjacent panel point. In space, a minimum of three strain lines

of known direction is necessary to locate, geometrically, the position of an adjacent panel point. Perpendicular planes rather than perpendicular lines are required to determine the location of a displaced joint for there is no assurance that the resulting intersection will fall in the original plane of any two strain lines.)

9. The continuation of the procedure of erecting strain lines from known or located joints; of constructing planes perpendicular to their ends, and of intersecting these planes.

10. The correction of the completed diagram—a rotational correction diagram is required provided an original assumption as to direction of any member is made.

Deflections in a Space Pedestal

In order to illustrate the method of construction involved in a three-dimensional displacement diagram, the simple space pedestal shown in Fig. 1 has been chosen as the first example. The frame has sixteen members and eight joints, four of which are points of application for reactions. The reactions are eight in number, four holding the pedestal on a horizontal datum plane and four maintaining equilibrium and stability within this horizontal plane.

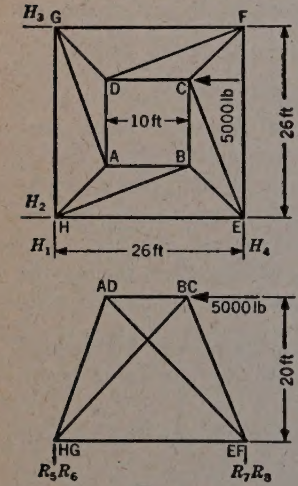

Fig. 1

A brief study will indicate that the frame is a statically determinate one because the twenty-four unknowns involved are not in excess of the available equations of statics. Although this particular quality is not a prerequisite for constructing a space displacement diagram, it is considered desirable for the primary illustration of the graphical method.

For the purpose of this example, the frame has been subjected to only one horizontal force of 5,000 lb, applied at the upper joint C. This simply gives rise to a number of inactive members—members carrying no axial load. Since this recourse has the effect of simplifying the graphical construction of the displacement diagram, it is advantageously used in the initial demonstration.

The space pedestal chosen has the added advantage of being fixed completely at one point and of having its lower horizontal members fixed in direction by the reactions. Thus, the necessity for a rotational correction diagram has been eliminated since no assumption of direction need be made for the original members meeting at the fixed point.

In Table 1, the axial forces in the various members are listed together with the $\frac{L}{A}$-values assumed for each member. (The axial force analysis can be found in the complete research on file in the Engineering Societies Library.[1a]) The deformation factors $P\,L/A$ are proportional only to the actual strains since the modulus of elasticity has been neglected. This device merely affords a

larger than actual value of deformation, which is plotted more readily in the space displacement diagram. Displacement values, scaled from the completed diagram, also will be proportional only to the actual values.

The Space Displacement Diagram for the Space Pedestal

The graphical procedure used in the construction of the three-dimensional diagram has been illustrated in three separate phases (see subsequently in Figs. 2, 3, and 4). The development of the diagram in progressive phases has been resorted to in order to clarify this initial illustration. The processes of descriptive geometry, inherent in a problem of this nature, are of temporary significance only and, consequently, have been omitted wherever possible in ensuing phases of the construction.

The notation employed in the three-dimensional displacement diagram is as follows:

TABLE 1.—Deformation Factors for the Space Pedestal, Fig. 1

Bar	P	L/A	PL/A
AB	0	2	0
BC	0	2	0
CD	−5,000	2	−10,000
DA	0	2	0
HE	0	5.2	0
EF	−1,540	5.2	− 8,000
FG	−3,460	5.2	−18,000
GH	+1,540	5.2	8,000
HA	0	4.54	0
HB	0	5.6	0
EB	0	4.54	0
EC	0	5.6	0
FC	0	4.54	0
FD	+5,390	5.6	+30,200
GD	−4,420	4.54	−20,050
GA	0	5.6	0

A^v, A^h = the final position of any displaced joint;

A^vb^v, A^hd^h, etc. = the strain lines plotted orthographically with space directions identical to those in the corresponding truss member;

H-trace of $A'b'$ = the horizontal trace of a plane $A'b'$ perpendicular to the strain line Ab;

Locus of A = all feasible positions of a displaced joint A prior to its definite location;

HV = the intersection between the horizontal and vertical projection planes;

x, y, and z = the coordinate axes of space; and

x_A = deflection of displaced joint A in the x-direction.

Procedure.—To any convenient scale, the plan and elevation views of the framed pedestal are drawn as shown in Fig. 2. These views are used to determine the slope of the strain lines in the displacement diagram.

Joint H is selected as the fixed reference point and separated in plan and elevation by an assumed HV-line. In all cases, unless otherwise specified, the plotting of any point, line, or plane is performed in both plan and elevation views.

The axial deformation Hg is plotted parallel to its corresponding member in the actual frame. The plotting is in the direction of the movement of the free end. No assumption has been made as to the direction of Hg since joints G, H, E, and F are held to a horizontal datum plane by vertical reactions. Furthermore, members HG and HE are fixed in direction on this horizontal plane. The g-end of the strain line Hg consequently becomes the final position of the displaced joint G.

The strain He is plotted in its direction, zero deformation in the bar HI causes point e to coincide with point H, locating the final position of th displaced joint E.

From joints G and E strains Gf and Ef are plotted in the direction of th movement of the free ends. Perpendicular lines are erected at the ends of thes strain lines and intersected to locate joint F. Joint F is the last point to remai on the horizontal datum plane.

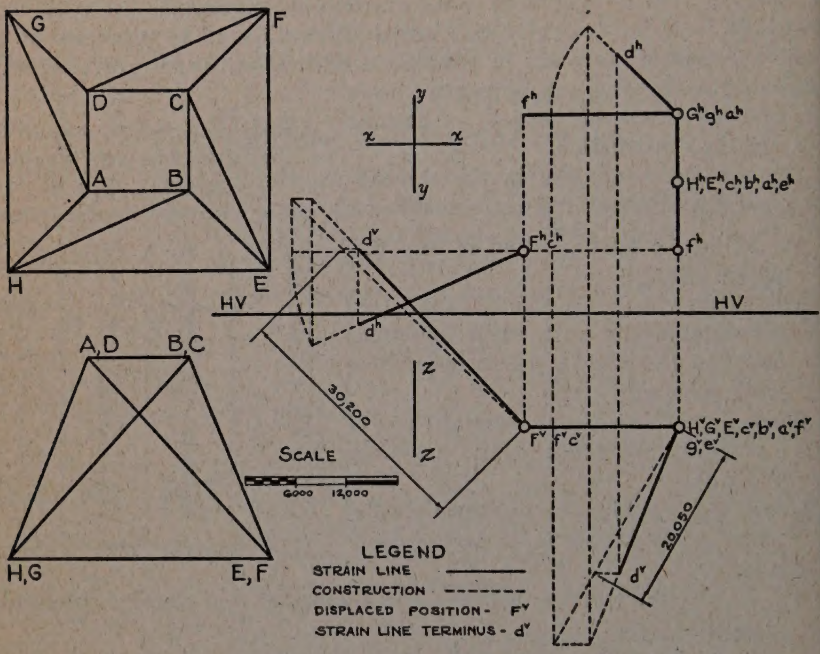

FIG. 2.—THREE-DIMENSIONAL DISPLACEMENT DIAGRAM, PHASE 1

Referring to the plan and elevation views of the actual pedestal, it is seen that from none of the joints thus far located can the final position of joints A B, C, or D be determined. This is evident since at least three members are required to fix any one point in space. All the space joints A, B, C, and D are positioned by only two members emanating from the basal joints H, G, E and F. The final position of any of these space joints is determined by its movement relative to another space joint.

Joint D is related to the base by members GD and FD. The locus of al possible positions of joint D is derived from the relationship between points D, G, and F. Since members GD and FD are three dimensional in direction the corresponding strain lines Gd and Fd are also three dimensional in direction The axial strains in GD and FD are of values +30,200 and −20,050, respectively, as seen from Table 1. These values are necessarily foreshortened in the displacement diagram representation since the actual frame members are foreshortened in both plan and elevation views. To plot these values correctly

it is necessary to find a true length projection of the proper strain line. For Fd and Gd this is done by revolving the horizontal projection of the lines to show true length in the vertical projection. On these true length projections, the true values of strain are plotted and then projected back into foreshortened plan and elevation views. This procedure locates the foreshortened strain lines Fd and Gd.

In similar fashion, the axial strains Ec and Fc are plotted parallel to their respective frame members. The strains in EC and FC are both zero, as indicated in Table 1, which causes c to coincide with joints E and F in the space diagram. Likewise, strains Eb, Hb, Ha, and Ga are zero and the free ends a and b coincide with the fixed joints E, H, and G.

Now, since all the strains rising from the basal points E, H, G, and F are three dimensional in nature, even though some are zero, perpendicular lines plotted at the free ends of the strain lines and presumably intersected, are of no significance. Rather, it is necessary to erect perpendicular planes to the ends of each of these strain lines and to intersect these planes. An intersection of two planes will yield a straight line which, in this case, will represent all possible positions of a displaced joint.

Fig. 3 shows the geometric procedure required for the erection of the eight planes perpendicular to the strain lines Gd, Fd, Ga, Ha, Hb, Eb, Ec, and Fc. The constructions required to plot foreshortened strain values for strains Gd and Fd have been omitted in Fig. 3 since they were of only temporary significance.

In order to erect the plane G′d′ perpendicular to the d-end of Gd, a horizontal line is drawn through point d perpendicular to the strain line Gd. This line, as indicated in Fig. 3, shows its perpendicularity in the horizontal projection and runs parallel to the HV-reference line in the vertical projection. The perpendicular line is continued in space until its horizontal projection intersects the HV-line, thus determining the vertical piercing point of the line. Through this vertical projection of the vertical piercing point the V-trace of the plane G′d′ is drawn perpendicular to the vertical projection of the strain line G^vd^v. The intersection of the vertical trace of G′d′ with the HV-reference line also indicates one point on the horizontal trace of G′d′. Through this intersection the H-trace of G′d′ is erected perpendicular to the horizontal projection of the strain line G^hd^h. The plane G′d′ represented by its horizontal and vertical traces is the required plane perpendicular to the strain line Gd at point d. Any descriptive geometry textbook will validate this method of construction.

In similar manner the plane F′d′ is erected perpendicular to the end of the strain line Fd, and is represented by its horizontal and vertical traces. The same method of construction is used to erect planes perpendicualr to the strain lines Ga, Ha, Hb, Eb, Ec, and Fc which are all equal to zero in length. In these cases, even though the strain lines are equal to zero in length, potential space directions are indicated by the plan and elevation views of the actual space pedestal.

These potential space directions must be recognized in order to erect the perpendicular lines which, together with the zero strain lines, are instrumental

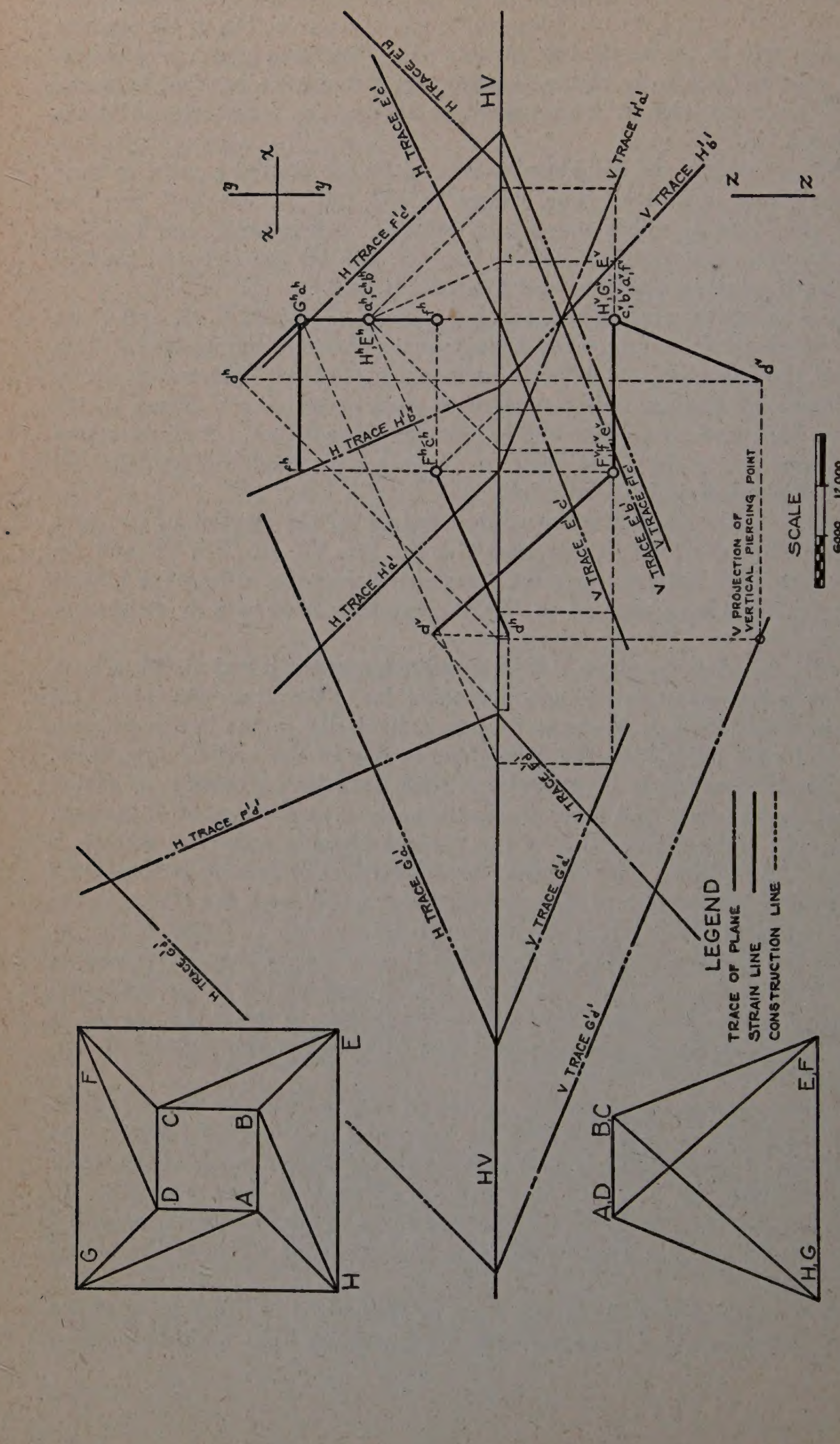

FIG. 3.—THREE-DIMENSIONAL DISPLACEMENT DIAGRAM, PHASE 2

in the construction of the planes. This completes the second phase of construction as shown in Fig. 3.

All dependence on the basal joints H, E, F, and G has now been reckoned with. There remains the determination of the loci of all feasible positions of joints A, B, C, and D and thence the location of their final positions.

In Fig. 4 it is evident that the intersection of the two planes G′d′ and F′d′ will indicate a line which serves as the locus of all feasible positions of point D. The locus of point D is found by realizing that the intersection of the H-traces of G′d′ and F′d′ and the intersection of the V-traces of the same planes locate two points on the line of intersection between the two planes. These two points are sufficient to determine the horizontal and vertical projections of the locus of point D. In like fashion, the locus of joint B is found in both horizontal and vertical projections.

The horizontal projections of the loci of joints A and C are found by locating the intersection of the H-traces of the proper planes. The vertical projection of the locus of A is then drawn parallel to the V-traces of planes G′a′ and H′a′ which are nonintersecting. In like manner the V-locus of joint C is drawn parallel to the V-traces of planes F′c′ and E′c′.

To find the final positions of joints A, B, C, and D, the relative movements of these space joints must be considered. As in the planar Williot diagram, the space diagram not only shows the movement of a particular joint with respect to its original position, but also indicates the movement of that joint with respect to any other joint.

From Table 1 it is seen that the following strains or deformations exist in the top horizontal members: AB = no deformation; AD = no deformation; DC = − 10,000 units of deformation; and CB = no deformation.

Since the relative x-displacement of joint A with respect to joint B is zero, joint A is located in the horizontal projection at the intersection of the H-locus of A with the H-locus of B. The final position of joint A in the vertical projection is obtained by projecting A^h perpendicular to the HV-reference line, to the V-locus of A. Point D is located in the horizontal projection at the intersection of the H-loci of A and D. This location is possible since the relative y-displacement of D with respect to A is zero. Joint D is located in the vertical plane by projecting D in the plane G′d′ to its proper position. The elements of descriptive geometry are necessary to effect this projection.

The relative x-movement between joints C and D is equal to 10,000 units. From the locus of D in the horizontal projection, point C is plotted along the H-locus of C, a distance of 10,000 units in the direction of the movement of the free end, C. Joint C is located in vertical plane by projecting a perpendicular from the HV-reference line to the V-locus of C.

At the intersection of the H-loci of C and B, the horizontal projection of joint B is located. This location is possible since the relative y-movement between C and B is equal to zero. Point B is then projected in the plane E′b′ to its proper location in the vertical view.

The space displacement diagram is thereby completed and the true positions of all joints without recourse to a rotational correction diagram are furnished. All values of displacement are scaled parallel to the coordinate axes x, y, and z

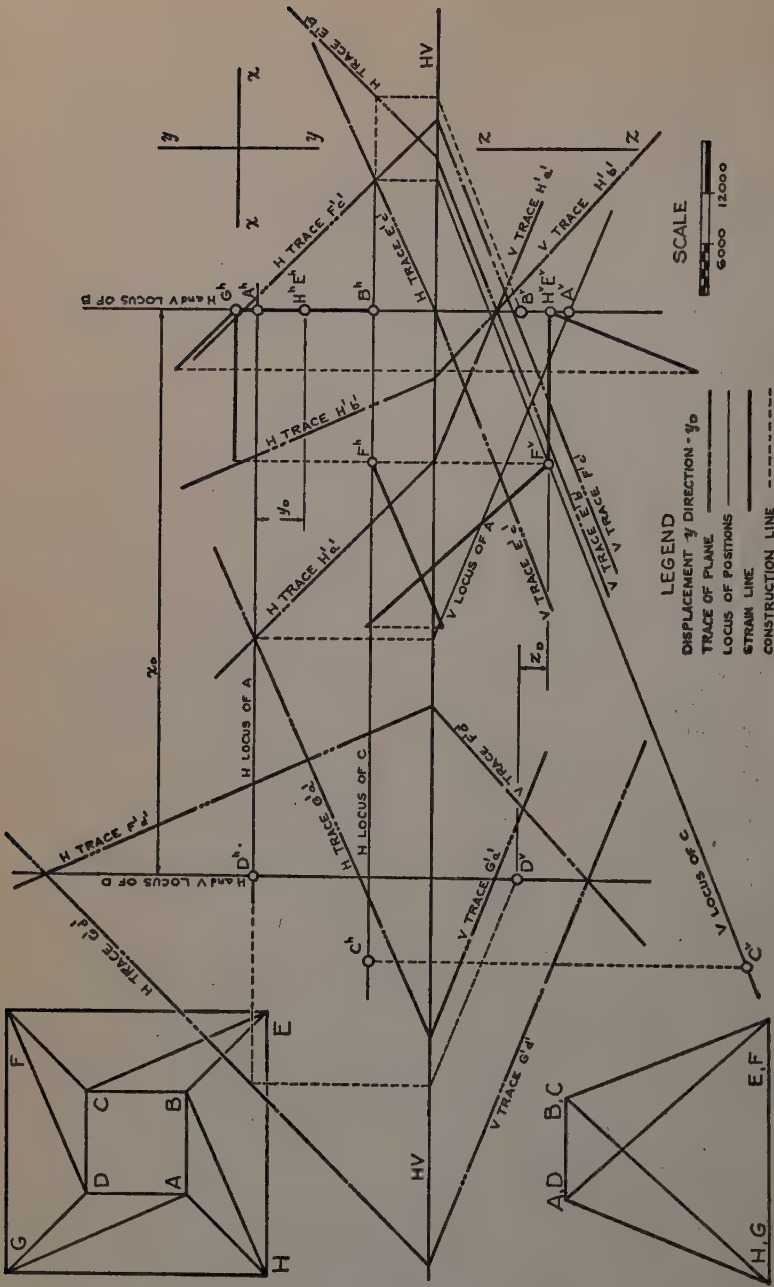

Fig. 4.—Three-Dimensional Displacement Diagram, Phase 3

from the fixed joint H. The values of the displacements x, y, and z of joint D from its original position are shown in Fig. 4 by x_D, y_D, and z_D.

To effect a check on the graphical procedure involved in the three-dimensional diagram, values of displacements x, y, and z were computed by the virtual work method as explained in the complete manuscript.[1a] These computations involved: (1) The solution for axial forces within the space pedestal under sixteen separate and distinctly different applications of unit load at the various joints; (2) the proper combination of these axial forces with the forces resulting from the 5,000-lb load; and (3) the algebraic summation of all $\frac{PuL}{AE}$-values pertinent to a particular displacement. The time required for the analytical method was at least tenfold that required for the graphical analysis. The values of displacement obtained graphically were within 1% of those calculated by the virtual work principles.

Deflections in a Space Truss

As a second and more generalized application of the principles involved in space displacement diagram construction, the deflections are found graphically for the space truss shown in Fig. 5.

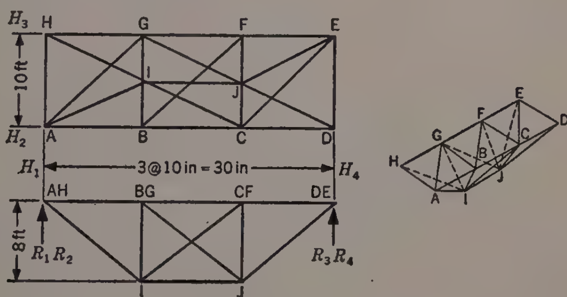

Fig. 5

This particular truss has been selected as an illustration because it embodies all the characteristics which ordinarily tend to complicate the construction of displacement diagrams. The construction of an accurate three-dimensional displacement diagram for this truss should accordingly indicate the applicability of the method to any type of space structure, however complicated.

Primarily, the space truss is an indeterminate one. The eight external reactions required to maintain horizontal and vertical stability consist of four horizontal and four vertical components. Furthermore, the truss consists of ten joints and twenty-four bars, none of which lie in a common plane at any of the ten joints. Applying the test for redundancy in space, it is observed that the number of unknown elements is in excess of the number of available static equations by two. Consequently, the space truss considered is statically indeterminate to the second degree.

The application of 10-kip, panel-point concentrations at each upper panel point is in keeping with the standard method of applying dead load. This

loading tends to reduce the number of inactive members to a minimum, which in turn has the effect of making the final displacement of any joint dependent upon a number of internal deformations. Such a dependence upon internal deformations necessitates the construction of strain lines for practically every member of the truss.

Although the truss has been loaded symmetrically with 10-kip concentrations, the nonsymmetry of the internal member and the reaction arrangement cause an inequality in axial force values on either side of the center line. This inequality, in turn, eliminates the possibility of choosing a member that will maintain a fixed direction under loading. If an assumption as to the direction of any member must be made at the outset of the construction, a rotational correction diagram must be incorporated within the construction in order to arrive at the true values of displacement. Since the displacement diagram for this space truss requires an initial assumption as to the direction of one of its chords, a rotational diagram is needed. This rotational diagram tends to generalize the construction further.

TABLE 2.—NUMERICAL EXAMPLE, FIG. 5

Bar	P	$P\,L/A$
AB	−13.60	−13.60
BC	−11.77	−11.77
CD	−11.22	−11.22
DE	− 5.06	− 5.06
EF	−11.95	−11.95
FG	−10.12	−10.12
GH	− 9.57	− 9.57
HA	− 5.61	− 5.61
AG	− 2.57	− 2.57
BG	+ 8.08	+ 8.08
BF	− 2.57	− 2.57
FC	+ 8.08	+ 8.08
CE	− 2.57	− 2.57
HI	+15.48	+15.48
AI	+18.95	+18.95
GI	−10.60	−10.60
BI	−11.80	−11.80
IJ	+25.28	+25.28
IC	− 1.75	− 1.75
JG	− 1.75	− 1.75
JF	−11.80	−11.80
JC	−10.60	−10.60
JE	+18.95	+18.95
JD	+15.48	+15.48

In Table 2 the axial forces in the various members of the indeterminate space truss are listed together with the resulting deformation factors ($P\,L/A$) in each member. The complete computation is included in the original manuscript on file in the Engineering Societies Library.[1a]

The $\frac{L}{A}$-term has been assumed unity for each member because of the facility it affords in indeterminate force computations. Although the resulting deformations and displacements are disproportionate under an assumption of a unit $\frac{L}{A}$-value for all members, none of the principles of stress analysis has been violated; nor have any of the difficulties in displacement diagram construction been circumvented. The modulus of elasticity has been neglected simply to yield larger values of deformation which are more easily plotted.

Subsequently, Fig. 6, 7, and 8 show, in three phases, the three-dimensional displacement diagram for the space truss. Since many of the internal members of the truss are parallel to the profile plane, it has been considered advisable to represent the diagram in profile projection as well as in horizontal and vertical projections.

The notation is essentially the same as was used for the space pedestal; any additional notation resulting from the use of a profile projection is self evident.

Procedure

To any convenient scale the space truss is represented orthographically in all three of the common views as shown in Fig. 6. All views are used to determine the corresponding slopes of the strain lines in the displacement diagram.

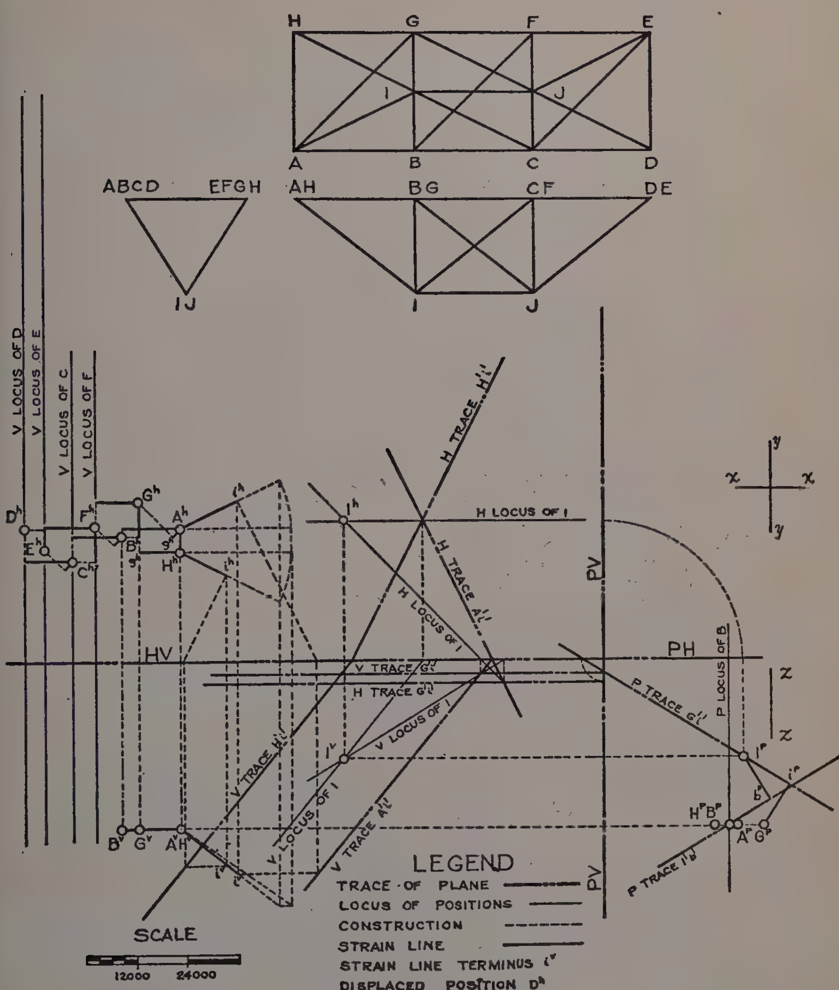

Fig. 6.—Three-Dimensional Displacement Diagram; Space Truss, Phase 1

Phase 1.—Since joint A is the only completely fixed joint in the truss, it is used as the origin of the displacement diagram. Joint A is plotted and separated in the three views by reference lines HV, PV, and PH.

Referring to the orthographic representation of the space truss it is seen that the truss consists of two upper chords, AD and HE, separated by lateral

bracing members, HA, GA, GD, etc., all of which lie in the same horizontal plane. As far as the x-direction and y-direction of space are concerned, the true displacements of all upper chord joints in this horizontal system may be determined by an ordinary Williot diagram. This is essentially the initial step. The final elevation of each of these joints is dependant upon the deformations in the lower chord and the connecting system. The final elevations may be determined at a later stage in the diagram development.

From the fixed joint A, the strain Ah is plotted in its true projection. Since there is no assumption as to the direction of Ah, point H is determined immediately. In the plan view only, point G is next located. The position of G^h is determined by intersecting planes set perpendicular to strain lines originating at A^h and H^h. Since both planes are vertical (both are erected perpendicular to horizontal strain lines), their line of intersection projects on the plan view as a point. This line of intersection represents the locus of all feasible positions of joint G, and definitely locates the x-component and the y-component of displacement of joint G. The elevation view of joint G depends on the strains in members IG and GJ and remains to be found.

From joint G in plan view, the strain in the member GB is plotted, and from joint A, the strain in member AB is plotted. Planes are set perpendicular to the ends of these strain lines and intersected to locate B^h. As in the case of joint G, the line of intersection of the planes locating B^h is a vertical line which projects as a point in the horizontal or plan view.

In similar fashion the loci of joints F, C, E, and D are found in the order indicated, making use of strains in only the horizontal top truss.

All x-components and y-components of displacement for joints A, B, C, D, E, F, G, and H are now ascertained.

In order to locate the vertical projections of the upper chord joints, and the horizontal and vertical projections of the lower chord joints I and J, the construction must become three dimensional in nature.

The Strain Lines Hg *and* Ag *Are Now Assumed to Be Horizontal, and the Vertical and Profile Positions of* G *Are Determined.*—This assumption is comparable to considering an initial chord member in a planar truss to be horizontal in order to reach the next panel point by planar Williot diagram construction. Point G, so located, obviously is in an incorrect position. Since the elevation view location of all ensuing joints is dependent upon the position of joint G, a rotational correction diagram will be necessary.

Strain lines Ai and Hi are erected in plan and elevation views. Each of these strain lines is foreshortened in orthographic projection. The strain lines are consequently revolved into true length views and true values of deformation are plotted on the true length views. The values of deformation are then projected back into the foreshortened horizontal and vertical views.

Planes H′i′ and A′i′ are erected perpendicular to these strain lines at their free ends. The planes are represented by their horizontal and vertical traces.

The strain in GI is plotted in the three-dimensional displacement diagram parallel to its corresponding truss member and the plane G′i′ is erected perpendicular to the end of Gi. This procedure is carried out most advantageously

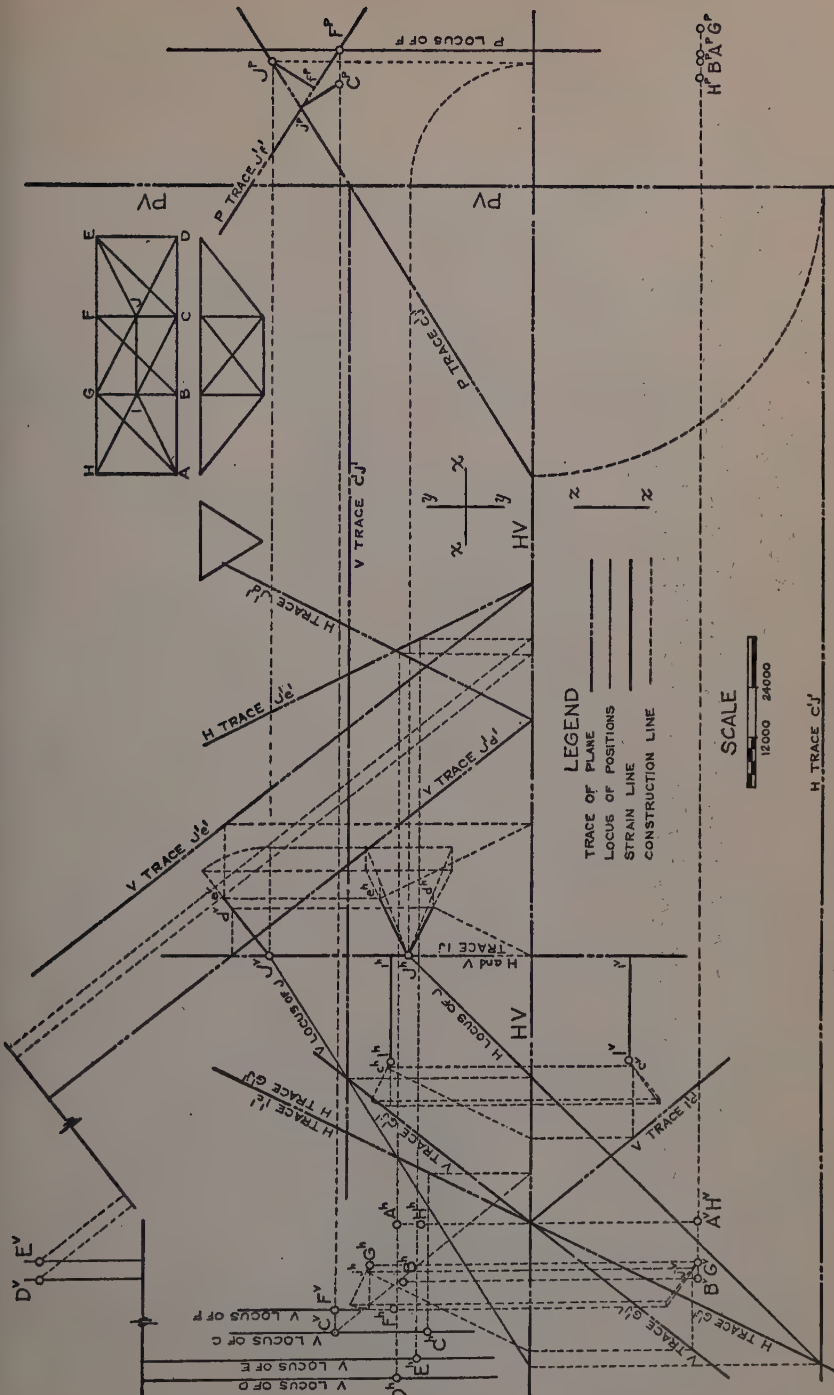

FIG. 7.—THREE-DIMENSIONAL DISPLACEMENT DIAGRAM; SPACE TRUSS, PHASE 2

in the profile view since member GI is parallel to the profile and, consequently, is projected in true length on this plane.

Planes A′i′ and H′i′ are intersected to find the locus of positions of joint I. Planes A′i′ and G′i′ are intersected to find a second locus of position for joint I. To accomplish this, plane G′i′ is shown by its traces on the horizontal and vertical planes of projection. The intersection of the two loci will determine the only possible position of joint I, which is then located in all three orthographic views. With the location of joint I now fixed, the process may be continued by locating joint B in its final position.

In the profile view the strain Ib is plotted parallel to the truss member and in the direction of the free end b. The plotting is done in the profile since IB projects in true length on the profile plane. Plane I′b′ is now erected perpendicular to I^pb^p at point b^p and is represented by its profile trace. The intersection of the plane I′b′ with the locus of B, shown in profile, fixes the position of joint B. The final position of B is shown in all three orthographic projections. This completes Phase 1 of the construction shown in Fig. 6.

Phase 2.—Phase 2 of the displacement diagram illustrated in Fig. 7 involves the determination of the vertical projection location of joints C, J, F, D, and E. All prior constructions required in the location of joints in Phase 1 have been omitted in Phase 2 to clarify the diagram.

With the position of joint I known, the location of joint C is a simple matter. In Fig. 7 the strain Ic is plotted in its foreshortened positions and the plane I′c′ is erected perpendicular to the free end c. The plane I′c′ is represented by it horizontal and vertical traces. Knowing the horizontal projection C^h of joint C and knowing that joint C must fall in the plane of I′c′, the vertical projection C^v is readily determined by an elementary process of descriptive geometry. From C^h a line is drawn parallel to HV until it intersects the horizontal trace of I′c′. The vertical projection of this same line is drawn parallel to the vertical trace of I′c′ until it intersects the V-locus of C. This intersection locates the point C^v.

The location of joint J is now determined from the strains resulting in members CJ, GJ, and IJ. The strain cj is plotted in the profile plane, where it appears in its true length, and the plane C′j′ is erected perpendicular to the j-end of Cj. The plane C′j′ is then represented by its horizontal and vertical traces. In the horizontal and vertical projections planes G′J′ and I′j′ are erected perpendicular to their respective strain lines Gj and Ij. Planes G′j′ and C′j′ are intersected and a locus of possible positions for point J is determined. The intersection of the locus of J with the plane I′j′ determines J^v and J^h.

The remaining construction shown in Phase 2 of the displacement diagram involves the final location of points F, D, and E in the order indicated. This order of procedure is dictated by the member arrangement in the actual truss. No one joint can be located completely without considering its dependence on the position of a previously located joint.

Phase 3.—This completes the space displacement diagram construction and locates the relative displacement of all joints. A rotational correction diagram

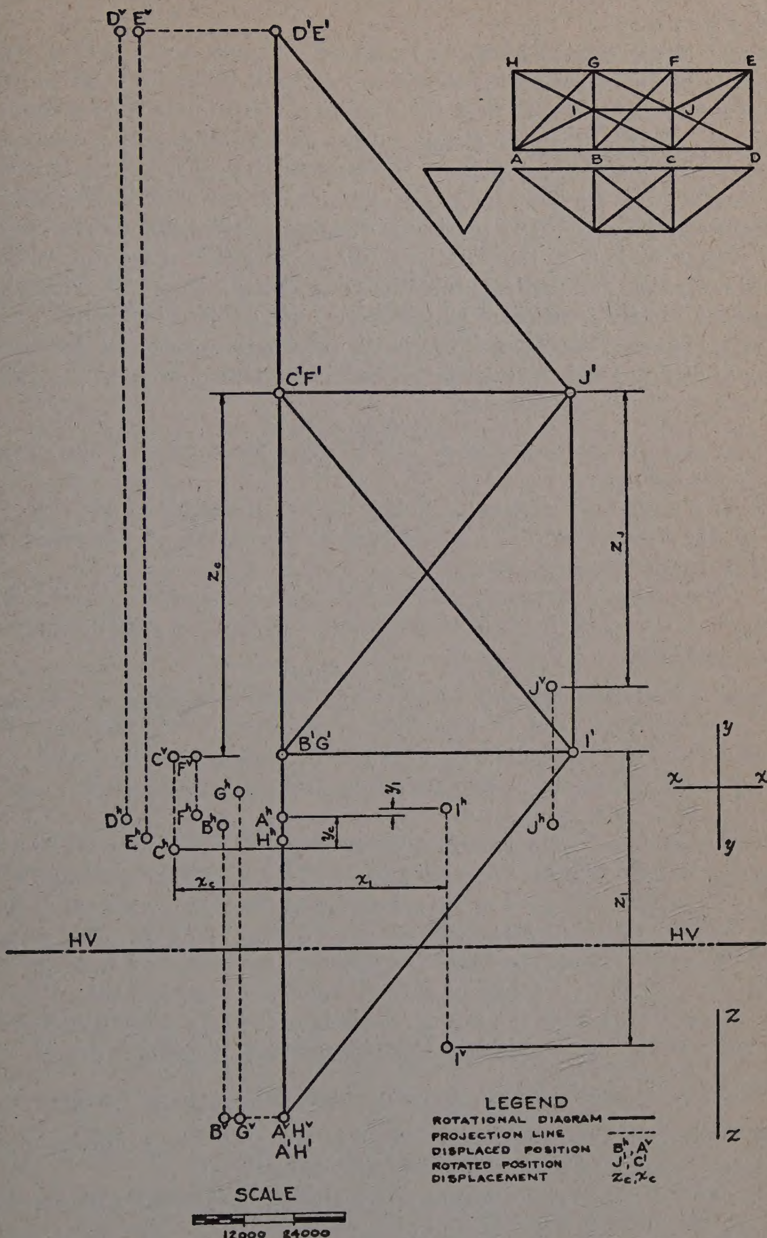

FIG. 8.—THREE-DIMENSIONAL DISPLACEMENT DIAGRAM; SPACE TRUSS, PHASE 3

is now required to eliminate the fallacies arising from the initial assumption of position for joint G.

It may be seen from Figs. 6 and 7 that the assumption of a vertical position of joint G has no effect on the x-displacement and the y-displacement of any joint, even though it does affect the z-position of all joints. As a result, the rotational correction diagram should correct for z-position only.

Obviously, if the space diagram represented true displacements of joints from their original position, the panel joints D and E would lie in a horizontal plane through joint A. The rotational diagram rectifies this shortcoming.

In Phase 3, Fig. 8, the rotational diagram is shown together with the located horizontal and vertical projections of all truss joints. The construction instrumental in the location of all joints has been omitted in Fig. 8.

For convenience the truss is left in its deflected position as determined by the space diagram and the undeflected correction truss is rotated through 90°. This may be effected as follows:

(*a*) A′ and H′ are located at A^v and H^v since the rotational movements of these joints are zero.

(*b*) E′ and D′ are located on a horizontal line through E^v and D^v and on a vertical line through A^vA'. This determines the length of the chords A′D′ and H′E′ in the rotational diagram.

(*c*) The complete elevation view of the truss is drawn to the scale of the top chord A′D′. This is the complete rotational diagram used for correcting vertical positions.

True vertical displacements are now obtained from the diagram by scaling distances A^vA', B^vB', and C^vC' parallel to the z-axis. Displacements in the x-direction and the y-direction are obtained by scaling distances, A^hC^h, A^hB^h, A^vC^v, etc., parallel to the coordinate axes x and y on the unaltered horizontal projection of the displacement diagram.

Because of the introduction of the correctional diagram in the vertical view, it was considered advisable to obtain a check of the values for vertical displacement. The classical method of virtual work was employed to obtain analytical values for the vertical displacement of joints B, C, F, and G. (For computation see complete research on file in the Engineering Societies Library.[1a]) The graphical and analytical values of deflection differ by no more than 1%. The concurrence of values by the two methods is necessarily dependent upon the degree of accuracy with which the three-dimensional diagram is drawn.

The Three-Dimensional Displacement Diagram in Retrospect

An analysis of the procedure involved in the preceding examples of space displacement can lead only to the following deductions:

The space displacement diagram, although closely akin to the planar Williot diagram, embodies a few differentiating characteristics. The same straightforwardness of attack employed in the planar Williot diagram cannot always be used in the three-dimensional construction. Ordinarily, the planar construction proceeds progressively joint by joint until the complete picture is obtained. In the usual space frame the joints are interdependent upon one

another for their exact location. This mutual relationship between joints eliminates the possibility of a step-by-step solution and dictates the use of a more circuitous method of attack. This deviousness is well illustrated by the first example of space diagram construction. The location of the upper panel points of the space pedestal cannot be fixed without recourse to a study of the relative movement between the upper joints. As long as the general stability of a space frame is contingent upon all its members, this roundabout type of solution cannot be avoided.

Other differences existing between the space and planar displacement diagram are minor. The use of perpendicular planes at the end of strain lines, in lieu of spherical surfaces, and the intersection of these planes to form loci of possible positions for joints, is in keeping with the reasoning employed by Williot. Proceeding from planar to spacial construction, the use of perpendicular planes rather than perpendicular lines follows as one of the natural consequences.

The use of a rotational correction diagram is not in itself a new idea, but its application to one or more views of a space displacement diagram might be considered unique.

Lastly, it may be deduced that any objection to the three-dimensional diagram on the basis of its complexity is groundless. The diagram is merely a sequence of superimposed constructions all of which are fundamental in character. If the clarity of the diagram is impaired, at any time, by previous constructions, all located points, loci of points, and required planes may be transferred to a new and lucid picture. This is possible because the processes of descriptive geometry used in such a problem are of temporary significance only.

In conclusion, it should be pointed out that the principles involved in three-dimensional displacement diagram construction can be used in the deflection study of any space frame.

This graphical method of finding displacements will not precipitate any strong inclinations toward space frame analysis, but it will aid in those solutions that must be made. The planar Williot diagram has been put to many uses other than that of finding deflection. Someday, the space displacement diagram may have a similar number of applications. The development of these applications will follow, in natural sequence, the desire to simplify the analysis of space structures further.